

Pooling natural catastrophe risks in a community

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Motivation

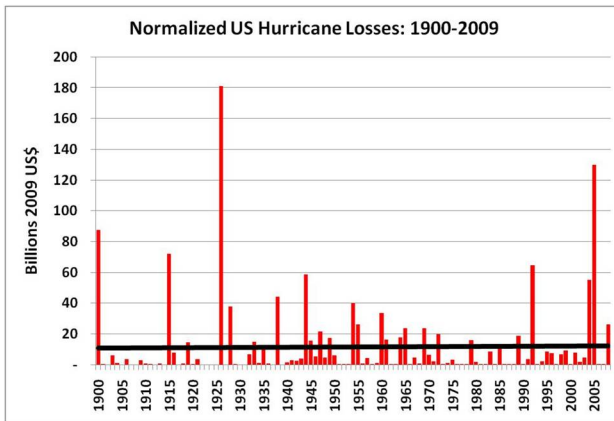
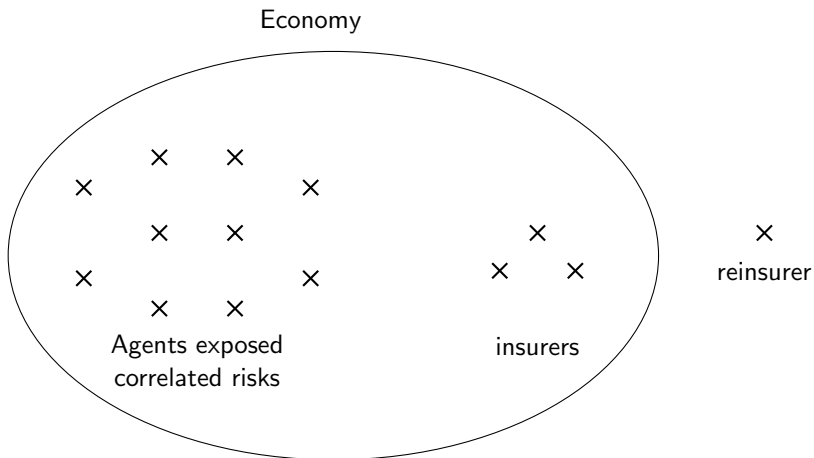


Figure: Normalized US Hurricane Losses (Pielke et al. (2008))

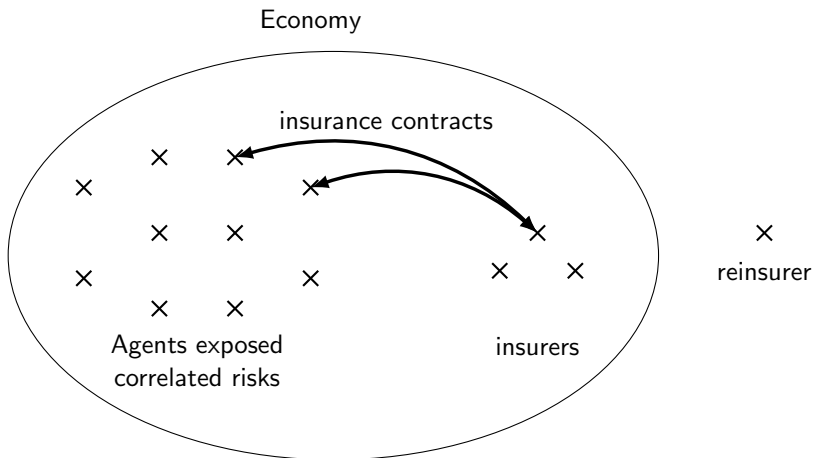
Questions

- ▶ In an economy with agents exposed to **correlated risks** and insurers facing **costly capital**:
- ▶ How are insurance contracts affected?
- ▶ How much do insurers default?
- ▶ Do agents invest in insurers' capital?

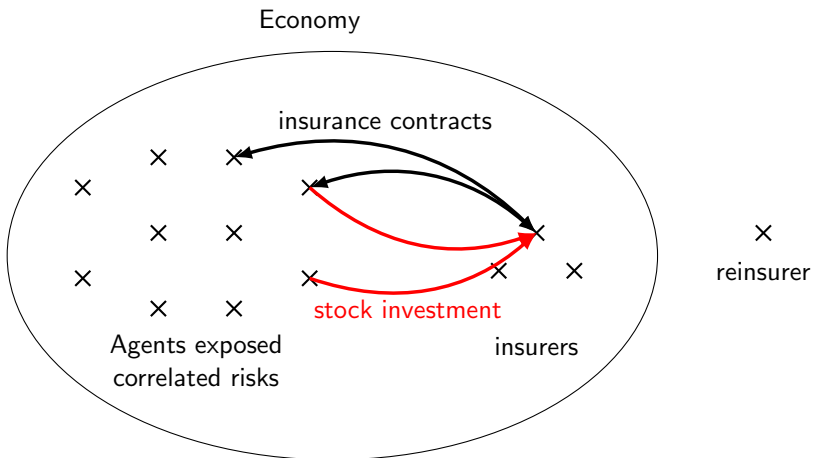
Framework



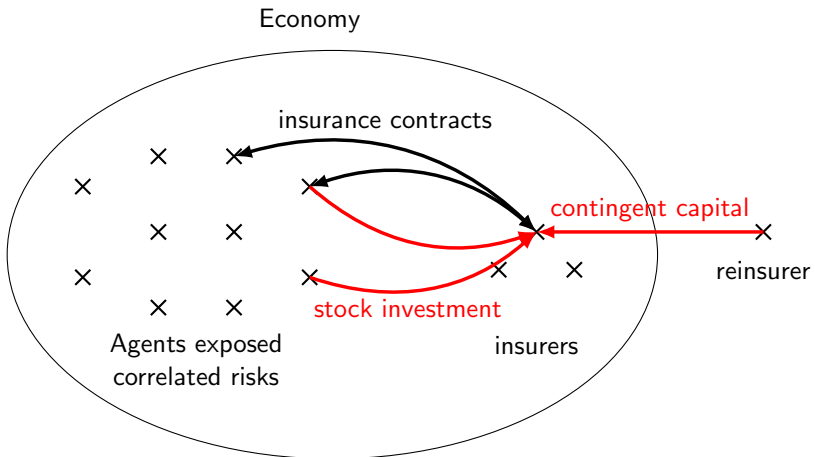
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Results

- ▶ With **transaction costs** in both the stock insurance market and the reinsurance market:
 - ▶ Agents purchase **full coverage**, but insurers **default** in catastrophic state.
 - ▶ Agents **invest in stock insurance if** and only if **stock insurance transaction costs are low** enough relative to reinsurance costs.

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 - ▶ Agents **invest in stock insurance if** and only if **stock insurance transaction costs are low** enough relative to reinsurance costs.
- ▶ Comparative statics relative to risk characteristics and transaction costs:
 - ▶ The more probable the catastrophe, the lower the default if agents invest in stock insurance.
 - ▶ The more costly the reinsurance, the lower the default if agents invest in stock insurance.

Related literature

- ▶ **No transaction cost** in the stock insurance market:
 - ▶ Borch (1962), Malinvaud (1973), Penalva-Zuasti (2001,2008),
 - ▶ Doherty and Dionne (1993), Doherty and Schlesinger (2002).
- ▶ **Infinite transaction cost** in the stock insurance market:
 - ▶ Doherty and Schlesinger (1990), Cummins and Mahul (2004,2007), Bernard and Ludwosky (2012),
 - ▶ Zhou et al. (2010), Biffis and Millossovich (2012), Charpentier and Le Maux (2014).

Risk-exposed agents

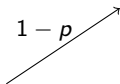
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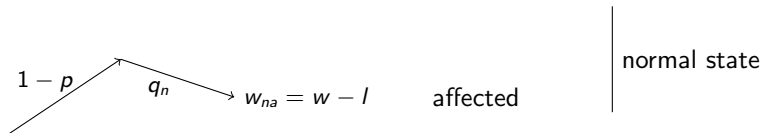
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normal state

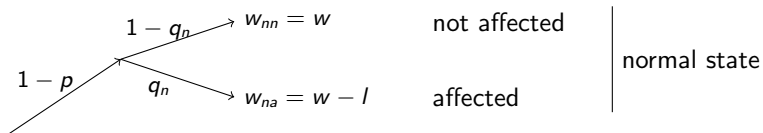
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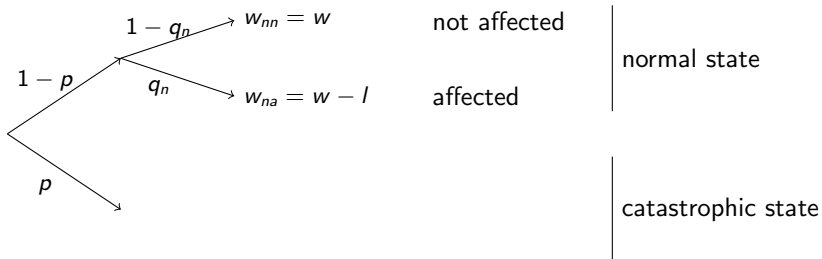
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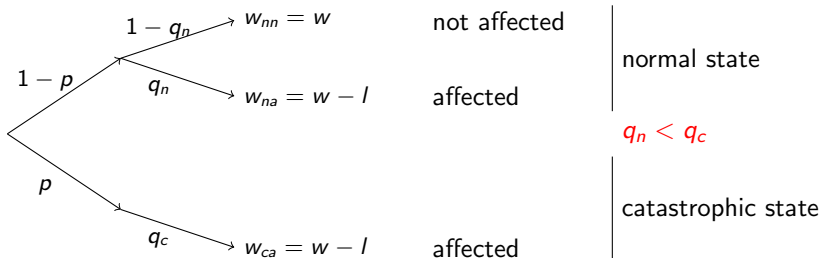
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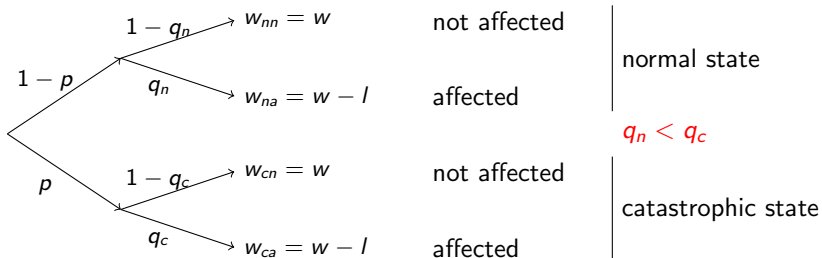
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Insurance companies

- ▶ Identical insurance companies in competition supply standard insurance contracts with potential default in the catastrophic state:
 - ▶ **premium** $\alpha(\tau, \epsilon)$ in exchange for **indemnity** τ for loss l with **default** ϵ in the catastrophic state.

Insurance companies

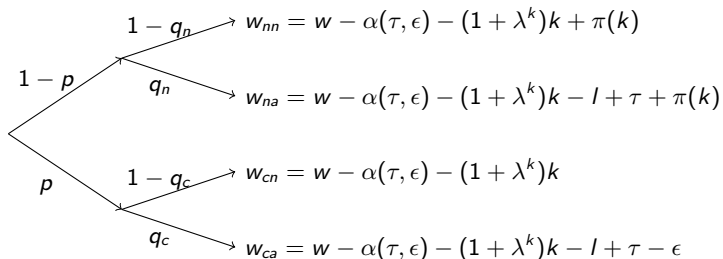
- ▶ Identical insurance companies in competition supply standard insurance contracts with potential default in the catastrophic state:
 - ▶ premium $\alpha(\tau, \epsilon)$ in exchange for indemnity τ for loss l with default ϵ in the catastrophic state.
- ▶ Insurance companies are owned by agents in the economy through stock markets:
 - ▶ capital $(1 + \lambda^k)k$ invested to get dividend $\pi(k)$ in the normal state, in which $0 < \lambda^k$ is the stock market transaction cost.

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- ▶ Insurance companies can purchase reinsurance outside the economy:
 - ▶ premium $(1 + \lambda^R)p\tau^R$ in exchange for indemnity τ^R , in which $0 < \lambda^R < \overline{\lambda^R}$ is the reinsurance loading factor.

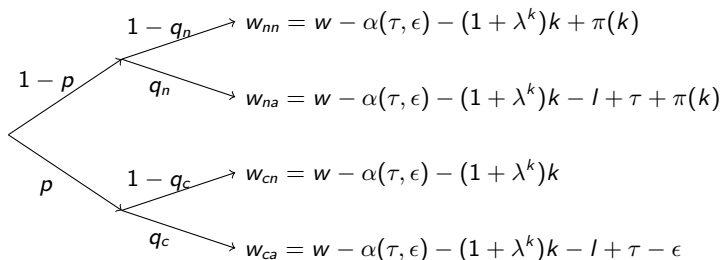
Agent and insurer wealth profiles

- ▶ The risk-exposed agent wealth profile is:

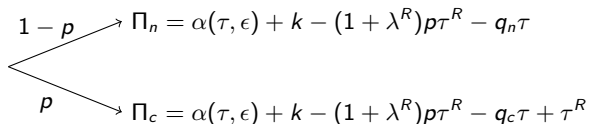


Agent and insurer wealth profiles

- ▶ The risk-exposed agent wealth profile is:



- ▶ The insurance company wealth profile (per agent) is:



Equilibrium

- ▶ Risk-exposed agents maximize their utility:

$$\begin{aligned} \max_{\tau, \epsilon, k} \mathbb{E}(u(\tilde{w})) \\ \text{s.t. } \tau \geq 0, \tau - \epsilon \geq 0 \text{ and } k \geq 0. \end{aligned} \quad (1)$$

- ▶ Positive and negative insurer profits correspond to dividends and default respectively:

$$\Pi_n = \pi(k) \text{ and } \Pi_c = -q_c \epsilon \quad (2)$$

First order conditions

- ▶ The indemnity demand τ , the default demand ϵ and the capital supply k are such that:

$$\underbrace{(1-p)q_n u'(w_{na}) + pq_c u'(w_{ca})}_{\text{marginal benefit from } \tau} = \underbrace{\frac{\partial \alpha}{\partial \tau} \mathbb{E}(u'(\tilde{w}))}_{\text{marginal cost from } \tau}$$

$$\underbrace{-\frac{\partial \alpha}{\partial \epsilon} \mathbb{E}(u'(\tilde{w}))}_{\text{marginal benefit from } \epsilon} = \underbrace{pq_c u'(w_{ca})}_{\text{marginal cost from } \epsilon}$$

$$\underbrace{(1-p)(1-q_n)u'(w_{nn}) + (1-p)q_n u'(w_{na})}_{\text{marginal benefit from } k} = \underbrace{\frac{1+\lambda^k}{\frac{\partial \pi}{\partial k}} \mathbb{E}(u'(\tilde{w}))}_{\text{marginal cost from } k} \text{ or } k=0$$

Prices

- ▶ The insurance premium $\alpha(\tau, \epsilon)$ is such that:

$$\alpha(\tau, \epsilon) = \underbrace{\left(1 + \frac{p(q_c - q_n)}{\bar{q}} \lambda^R\right)}_{<1+\lambda^R} \bar{q}\tau - \underbrace{\left(1 + \lambda^R\right)}_{>1} pq_c\epsilon$$

- ▶ The insurance dividend $\pi(k)$ is such that:

$$(1 - p)\pi(k) = \frac{1}{\underbrace{1 - \frac{p}{1-p} \lambda^R}_{>1}} k$$

Equilibrium

Result 1:

Equilibrium is characterized by:

- (i) **insurance contracts with full coverage** ($\tau = 1$).
- (ii) **insurers default** ($\epsilon > 0$) iff $\lambda^K > 0$ and $\lambda^R > 0$.
- (iii) **investment in stock insurance markets** ($k > 0$) iff $\lambda^k < \lambda^{k*}(\lambda^R)$.

Comparative statics relative to risk characteristics

Result 2:

With a CARA utility function, we have:

(i) for a larger catastrophe (i.e. q_c):

(a) $\lambda^k > \lambda^{k^*}$: $\frac{d\alpha}{dq_c}$ ambiguous, $\frac{d\epsilon}{dq_c} > 0$.

(b) $0 < \lambda^k < \lambda^{k^*}$: $\frac{d\alpha}{dq_c}$ ambiguous, $\frac{d\epsilon}{dq_c} > 0$, $\frac{d\pi}{dq_c} < 0$.

(ii) for a more probable catastrophe (i.e. p):

(a) $\lambda^k > \lambda^{k^*}$: $\frac{d\alpha}{dp}$ ambiguous, $\frac{d\epsilon}{dp} > 0$.

(b) $0 < \lambda^k < \lambda^{k^*}$: $\frac{d\alpha}{dp} > 0$, $\frac{d\epsilon}{dp} < 0$, $\frac{d\pi}{dp} > 0$.

Comparative statics relative to transaction costs

Result 3:

With a CARA utility function, we have:

(i) for a larger reinsurance loading (i.e. λ^R):

(a) $\lambda^k > \lambda^{k*}$: $\frac{d\alpha}{d\lambda^R}$ ambiguous, $\frac{d\epsilon}{d\lambda^R} > 0$.

(b) $0 < \lambda^k < \lambda^{k*}$: $\frac{d\alpha}{d\lambda^R} > 0$, $\frac{d\epsilon}{d\lambda^R} < 0$, $\frac{d\pi}{d\lambda^R} > 0$.

(ii) for a larger stock insurance transaction cost (i.e. λ^k):

$\frac{d\alpha}{d\lambda^k} < 0$, $\frac{d\epsilon}{d\lambda^k} > 0$, $\frac{d\pi}{d\lambda^k} < 0$.

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- ▶ We show:
 - ▶ Agents purchase full coverage, insurers default in catastrophic state and agents invest in stock insurance if not too costly.
 - ▶ The more probable the catastrophe or the more costly the reinsurance, the lower the default if agents invest in stock insurance.

Conclusion

- ▶ We develop a simple and tractable model with agents exposed to correlated risks and insurers having access to costly capital.
- ▶ We show:
 - ▶ Agents purchase full coverage, insurers default in catastrophic state and agents invest in stock insurance if not too costly.
 - ▶ The more probable the catastrophe or the more costly the reinsurance, the lower the default if agents invest in stock insurance.
- ▶ **Policy recommendation:** these last results suggest that policy makers should not necessarily authorize larger defaults with a more probable catastrophe or a more costly reinsurance.