Pooling natural catastrophe risks in a community

Arnaud Goussebaïle
joint work with Alexis Louaas

Actinfo Chair, Institut Louis Bachelier, CREST-Polytechnique

PET Conference
July 12th, 2017
Motivation

Figure: Normalized US Hurricane Losses (Pielke et al. (2008))
Questions

- In an economy with agents exposed to correlated risks and insurers facing costly capital:
  - How are insurance contracts affected?
  - How much do insurers default?
  - Do agents invest in insurers’ capital?
Framework

Economy

Agents exposed correlated risks

insurers

reinsurer

Introduction

Model

Results

Conclusion

Pooling natural catastrophe risks in a community
Pooling natural catastrophe risks in a community

Economy

Agents exposed correlated risks

 insurers

insurance contracts

reinsurer
Agents exposed correlated risks

Economy

insurance contracts

stock investment

insurers

reinsurer

Arnaud Goussebaïé

Actinfo Chair, Institut Louis Bachelier, CREST-Polytechnique

Pooling natural catastrophe risks in a community
Pooling natural catastrophe risks in a community
With transaction costs in both the stock insurance market and the reinsurance market:

- Agents purchase full coverage, but insurers default in catastrophic state.
- Agents invest in stock insurance if and only if stock insurance transaction costs are low enough relative to reinsurance costs.
Results

- With transaction costs in both the stock insurance market and the reinsurance market:
  - Agents purchase full coverage, but insurers default in catastrophic state.
  - Agents invest in stock insurance if and only if stock insurance transaction costs are low enough relative to reinsurance costs.

- Comparative statics relative to risk characteristics and transaction costs:
  - The more probable the catastrophe, the lower the default if agents invest in stock insurance.
  - The more costly the reinsurance, the lower the default if agents invest in stock insurance.
Related literature

- **No transaction cost** in the stock insurance market:

- **Infinite transaction cost** in the stock insurance market:
  - Doherty and Schlesinger (1990), Cummins and Mahul (2004, 2007),
    Bernard and Ludwosky (2012),
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function $u(.)$, wealth $w$ and risk of loss $l$. 

---

Arnaud Goussebaïle
Actinfo Chair, Institut Louis Bachelier, CREST-Polytechnique
Pooling natural catastrophe risks in a community
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function \( u(.) \), wealth \( w \) and risk of loss \( l \).

- Risks with two individual states and two collective states:
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function $u(.)$, wealth $w$ and risk of loss $l$.

- Risks with two individual states and two collective states:

  \[
  1 - p \
  \]

  normal state
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function $u(.)$, wealth $w$ and risk of loss $l$.

- Risks with two individual states and two collective states:

$$1 - p \quad q_n \quad w_{na} = w - l \quad \text{affected}$$

$1 - q_n < q_c$
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function $u(.)$, wealth $w$ and risk of loss $l$.

- Risks with two individual states and two collective states:

\[
\begin{align*}
1 - q_n & \quad w_{nn} = w & \text{not affected} \\
q_n & \quad w_{na} = w - l & \text{affected}
\end{align*}
\]
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function $u(.)$, wealth $w$ and risk of loss $l$.

- Risks with two individual states and two collective states:

  $1 - q_n \rightarrow w_{nn} = w$ not affected
  
  $1 - p \rightarrow q_n \rightarrow w_{na} = w - l$ affected
  
  $p \rightarrow$ catastrophic state
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function \( u(\cdot) \), wealth \( w \) and risk of loss \( l \).

- Risks with two individual states and two collective states:

\[
\begin{align*}
1 - q_n & \quad w_{nn} = w & \text{not affected} \\
1 - p & \quad q_n & \quad w_{na} = w - l & \text{affected} \\
p & \quad q_c & \quad w_{ca} = w - l & \text{affected} \\
q_n & \quad q_c & \quad w_{cn} = w & \text{not affected}
\end{align*}
\]

\( q_n < q_c \)
Risk-exposed agents

- An economy with identical risk-averse agents, with VNM utility function $u(.)$, wealth $w$ and risk of loss $l$.

- Risks with two individual states and two collective states:

  | $1 - q_n$ | $w_{nn} = w$ | not affected | normal state |
  | $q_n$ | $w_{na} = w - l$ | affected | $q_n < q_c$ |
  | $p$ | $w_{cn} = w$ | not affected | catastrophic state |
  | $1 - q_c$ | $w_{ca} = w - l$ | affected |
Insurance companies

- Identical insurance companies in competition supply standard insurance contracts with potential default in the catastrophic state:
  - premium $\alpha(\tau, \epsilon)$ in exchange for indemnity $\tau$ for loss $l$ with default $\epsilon$ in the catastrophic state.
Insurance companies

- Identical insurance companies in competition supply standard insurance contracts with potential default in the catastrophic state:
  - premium $\alpha(\tau, \epsilon)$ in exchange for indemnity $\tau$ for loss $l$ with default $\epsilon$ in the catastrophic state.

- Insurance companies are owned by agents in the economy through stock markets:
  - capital $(1 + \lambda^k)k$ invested to get dividend $\pi(k)$ in the normal state, in which $0 < \lambda^k$ is the stock market transaction cost.
Insurance companies

- Identical insurance companies in competition supply standard insurance contracts with potential default in the catastrophic state:
  - **premium** $\alpha(\tau, \epsilon)$ in exchange for **indemnity** $\tau$ for loss $l$ with default $\epsilon$ in the catastrophic state.

- Insurance companies are owned by agents in the economy through stock markets:
  - capital $(1 + \lambda^k)k$ invested to get **dividend** $\pi(k)$ in the normal state, in which $0 < \lambda^k$ is the stock market transaction cost.

- Insurance companies can purchase reinsurance outside the economy:
  - **premium** $(1 + \lambda^R)p\tau^R$ in exchange for **indemnity** $\tau^R$, in which $0 < \lambda^R < \overline{\lambda^R}$ is the reinsurance loading factor.
Agent and insurer wealth profiles

The risk-exposed agent wealth profile is:

\[
\begin{align*}
1 - q_n & \quad w_{nn} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k + \pi(k) \\
1 - p & \quad q_n \quad w_{na} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k - l + \tau + \pi(k) \\
p & \quad 1 - q_c \quad w_{cn} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k \\
q_c & \quad w_{ca} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k - l + \tau - \epsilon
\end{align*}
\]
Agent and insurer wealth profiles

- The risk-exposed agent wealth profile is:

  \[ w_{nn} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k + \pi(k) \]
  \[ w_{na} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k - l + \tau + \pi(k) \]
  \[ w_{cn} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k \]
  \[ w_{ca} = w - \alpha(\tau, \epsilon) - (1 + \lambda^k)k - l + \tau - \epsilon \]

- The insurance company wealth profile (per agent) is:

  \[ \Pi_n = \alpha(\tau, \epsilon) + k - (1 + \lambda^R)p \tau^R - q_n\tau \]
  \[ \Pi_c = \alpha(\tau, \epsilon) + k - (1 + \lambda^R)p \tau^R - q_c\tau + \tau^R \]
Equilibrium

- Risk-exposed agents maximize their utility:

\[
\max_{\tau, \epsilon, k} \mathbb{E}(u(\tilde{w})) \quad \text{s.t. } \tau \geq 0, \quad \tau - \epsilon \geq 0 \quad \text{and} \quad k \geq 0.
\]

(1)

- Positive and negative insurer profits correspond to dividends and default respectively:

\[
\Pi_n = \pi(k) \quad \text{and} \quad \Pi_c = -q_c \epsilon
\]

(2)
First order conditions

The indemnity demand $\tau$, the default demand $\epsilon$ and the capital supply $k$ are such that:

\[
(1 - p)q_n u'(w_{na}) + pq_c u'(w_{ca}) = \frac{\partial \alpha}{\partial \tau} \mathbb{E}(u'(\tilde{w}))
\]

marginal benefit from $\tau$
marginal cost from $\tau$

\[
- \frac{\partial \alpha}{\partial \epsilon} \mathbb{E}(u'(\tilde{w})) = pq_c u'(w_{ca})
\]

marginal benefit from $\epsilon$
marginal cost from $\epsilon$

\[
(1 - p)(1 - q_n) u'(w_{nn}) + (1 - p)q_n u'(w_{na}) = \frac{1 + \lambda^k}{\frac{\partial \pi}{\partial k}} \mathbb{E}(u'(\tilde{w})) \text{ or } k = 0
\]

marginal benefit from $k$
marginal cost from $k$
The insurance premium $\alpha(\tau, \epsilon)$ is such that:

$$\alpha(\tau, \epsilon) = \left(1 + \frac{p(q_c - q_n)}{q} \lambda^R\right) q^\tau - \left(1 + \lambda^R\right) p q_c \epsilon$$

The insurance dividend $\pi(k)$ is such that:

$$(1 - p)\pi(k) = \frac{1}{1 - \frac{p}{1 - p} \lambda^R} k$$
Result 1:

Equilibrium is characterized by:

(i) insurance contracts with full coverage ($\tau = l$).
(ii) insurers default ($\epsilon > 0$) iff $\lambda^K > 0$ and $\lambda^R > 0$.
(iii) investment in stock insurance markets ($k > 0$) iff $\lambda^k < \lambda^k*(\lambda^R)$.
Comparative statics relative to risk characteristics

Result 2:

With a CARA utility function, we have:

(i) for a larger catastrophe (i.e. $q_c$):
   
   (a) $\lambda^k > \lambda^{k^*}$: $\frac{d\alpha}{dq_c}$ ambiguous, $\frac{d\epsilon}{dq_c} > 0$.
   
   (b) $0 < \lambda^k < \lambda^{k^*}$: $\frac{d\alpha}{dq_c}$ ambiguous, $\frac{d\epsilon}{dq_c} > 0$, $\frac{d\pi}{dq_c} < 0$.

(ii) for a more probable catastrophe (i.e. $p$):

   (a) $\lambda^k > \lambda^{k^*}$: $\frac{d\alpha}{dp}$ ambiguous, $\frac{d\epsilon}{dp} > 0$.

   (b) $0 < \lambda^k < \lambda^{k^*}$: $\frac{d\alpha}{dp} > 0$, $\frac{d\epsilon}{dp} < 0$, $\frac{d\pi}{dp} > 0$. 

Arnaud Goussebaïe
Actinfo Chair, Institut Louis Bachelier, CREST-Polytechnique

Pooling natural catastrophe risks in a community
Comparative statics relative to transaction costs

Result 3:

With a CARA utility function, we have:

(i) for a larger reinsurance loading (i.e. $\lambda^R$):
   (a) $\lambda^k > \lambda^k^*$: \( \frac{d\alpha}{d\lambda^R} \) ambiguous, \( \frac{d\epsilon}{d\lambda^R} > 0 \).
   (b) $0 < \lambda^k < \lambda^k^*$: \( \frac{d\alpha}{d\lambda^R} > 0 \), \( \frac{d\epsilon}{d\lambda^R} < 0 \), \( \frac{d\pi}{d\lambda^R} > 0 \).

(ii) for a larger stock insurance transaction cost (i.e. $\lambda^k$):
   \( \frac{d\alpha}{d\lambda^k} < 0 \), \( \frac{d\epsilon}{d\lambda^k} > 0 \), \( \frac{d\pi}{d\lambda^k} < 0 \).
We develop a simple and tractable model with agents exposed to correlated risks and insurers having access to costly capital.
Conclusion

- We develop a simple and tractable model with agents exposed to correlated risks and insurers having access to costly capital.

- We show:
  - Agents purchase full coverage, insurers default in catastrophic state and agents invest in stock insurance if not too costly.
  - The more probable the catastrophe or the more costly the reinsurance, the lower the default if agents invest in stock insurance.
Conclusion

- We develop a simple and tractable model with agents exposed to correlated risks and insurers having access to costly capital.

- We show:
  - Agents purchase full coverage, insurers default in catastrophic state and agents invest in stock insurance if not too costly.
  - The more probable the catastrophe or the more costly the reinsurance, the lower the default if agents invest in stock insurance.

- Policy recommendation: these last results suggest that policy makers should not necessarily authorize larger defaults with a more probable catastrophe or a more costly reinsurance.