The role of insurance companies in a risky economy

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Motivation

Example: a simple economy composed of $7 \times 10^9$ people, each one exposed to an endowment risk distribution with 11 possible states (endowment from 0 to 10 units).
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The main issues are that:

- it requires to have a tremendous number of security markets.
- it requires to make public the individual state of each person in the economy.
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- The main issues are that:
  - it requires to have a tremendous number of security markets.
  - it requires to make public the individual state of each person in the economy.

- What role is played by insurance companies?
I show that Pareto optimality is reached with only $10 \times 7 \times 10^9 + 1$ of these security markets if people have also access to standard insurance contracts supplied by stock insurance companies.
Contribution

- I show that Pareto optimality is reached with only $10 \times 7 \times 10^9 + 1$ of these security markets if people have also access to standard insurance contracts supplied by stock insurance companies.

- The presence of insurance companies allow:
  - to reduce the required number of security markets by many orders of magnitude.
  - to lower tremendously the required public information.
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- to reduce the required number of security markets by many orders of magnitude.
- to lower tremendously the required public information.

I show this result in a static exchange economy with:

- multiple commodities,
- heterogeneous agents in terms of preferences and risk distributions,
- no restriction on risk dependence.
  - one commodity, heterogeneous agents, no restriction on risk dependence.
  - they do not explain who supplies insurance contracts.

  - one commodity, homogeneous agents, i.i.d. risks.
  - mutual insurance companies.

  - one commodity, heterogeneous agents, i.i.d. risks.
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Literature review

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  - Risky economy.
  - Spot and security markets.
  - Insurance companies.

Introduction

The model

The equilibrium

Conclusion

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  ▶ Spot and security markets.
  ▶ Insurance companies.

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Risky economy

- Exchange economy with $C$ commodities and $N$ risk-averse heterogeneous agents:
  - VNM utility for agent $i$: $v_i(.) : \mathbb{R}_+^C \to \mathbb{R}$
  - Endowment vector for agent $i$: $e_i(s_i) = \bar{e}_i - l_i(s_i)$, $s_i = 1, \ldots, S_i$
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**Definition of an Arrow-Debreu state**

An "Arrow-Debreu" state is a full specification of the individual endowments obtained by all the agents in the economy.
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Definition of an Fundamental state

A ”fundamental” state is a full specification of the aggregate endowments in the economy.
Risky economy

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Fundamental states: \( t(.) : [1, Z] \rightarrow [1, T] \)

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- With $F_t$ the set of Arrow-Debreu states in the Fundamental state $t$: $\forall z \in F_t, \sum_i e_i(s_i(z)) = \mathcal{E}(t(z))$
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- Consumption plan of agent \( i \) in Arrow-Debreu state \( z \) denoted: \( x_i(z) \).
Spot and security markets

- C spot markets:
  - after the state of nature has been revealed.
  - price vector: $p(z)$.
Spot and security markets

- **C** spot markets:
  - after the state of nature has been revealed.
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- **$T$** security markets:
  - security market $t$ enables to get one ex-post money unit if $t$ occurs, in exchange for $\sum_{z \in F_t} \pi(z)$ ex-ante money unit.
  - quantity of securities purchased/sold by agent $i$ denoted $a_i = (a_i(1), .., a_i(T))$
Insurance companies

- $M$ insurance companies in competition supply insurance contracts and are owned through stock markets.
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- An insurance contract for agent $i$ consists in:
  - an indemnity in state $z$:
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    \tau_i(p(z), s_i(z)) = p(z)l_i(s_i(z));
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  - in exchange for a premium in any state $z$:
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  - Quantity of insurance purchased by agent $i$: $n_i$. 

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- Profit of insurer $k$:
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  \]

- Share of insurer $k$ purchased by agent $i$: $m_{ki}$. 
Equilibrium

Agent maximization problem:

\[
\max_{x_i, n_i, m_{ki}, a_i} \sum_z \pi(z)v_i(x_i(z))
\]

s.t.

\[
\begin{align*}
\mathbf{p}(z)x_i(z) &= \mathbf{p}(z)e_i(s_i(z)) + \tau_i(\mathbf{p}(z), s_i(z))n_i \\
&\quad + \sum_k r_k(z)m_{ki} + a_i(t(z)), \quad \forall z \\
\alpha_i n_i + \sum_z \pi(z)a_i(t(z)) &= 0
\end{align*}
\]
Equilibrium

Agent maximization problem:

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s.t. \[ p(z)x_i(z) = p(z)e_i(s_i(z)) + \tau_i(p(z), s_i(z))n_i \]

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\[ \alpha_i n_i + \sum_{z} \pi(z)a_i(t(z)) = 0 \]

Market clearing conditions (spot, security, stock):

\[ \sum_{i} x_i(z) = E(t(z)), \ \forall \ z \in Z \]

\[ \sum_{i} a_i(t) = 0, \ \forall \ t \in T \]

\[ \sum_{i} m_{ki} = 1, \ \forall \ k \in M \]
Pareto optimality

Result

In the decentralized economy with $C$ spot markets, $T$ security markets and insurance companies in competition, the equilibrium exists and the allocation is Pareto optimal.
Role of insurance companies

- With insurance companies, it is sufficient to have $T$ security markets:
  - much lower quantity of security markets.
  - need to make public only the aggregate endowment.
Role of insurance companies

- With insurance companies, it is sufficient to have $T$ security markets:
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  - need to make public only the aggregate endowment.

- One way to reach her consumption plan for each agent $i$ is:
  - elimination of individual endowment risk with insurance contract: $n_i = 1$.
  - elimination of price risk exposure and participation to aggregate risk through security and insurance stock markets:
    \[
    a_i(t(z)) = p(z)(x_i(z) - \bar{e}_i) + \alpha_i - \frac{1}{N} \sum_k r_k(z) \text{ and } m_{ki} = \frac{1}{N}.
    \]
Insurance premiums

- Insurance companies are constrained to sell fair contracts to catch policyholders on one side and shareholders on the other side.
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- Insurance premium:

\[ \alpha_i = \sum_{z'} \pi(z')p(z')l_i(s_i(z')) = (1 + \beta_i)\bar{p} \sum_{z'} \pi(z')l_i(s_i(z')) , \]

- in which:

\[ \bar{p} = \sum_{z'} \pi(z')p(z') \]

\[ \beta_i = \frac{1}{\bar{p} \sum_{z'} \pi(z')l_i(s_i(z'))} \sum_{z'} \pi(z')(p(z') - \bar{p})l_i(s_i(z')) . \]
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- For an insurer, the higher the correlation between insured individual risks and the aggregate risk, the higher the expected profit.
Conclusion

- An exchange economy with multiple commodities, heterogeneous agents and no restriction on risk dependence.
- Only one security market per fundamental state is sufficient if there are also stock insurance companies supplying standard insurance contracts.
- Insurance and security markets respectively allow to deal with endowment risks and price risk.
- Insurance premium has an aggregate risk loading factor which is specific to each agent.