

The role of insurance companies in a risky economy

Arnaud Goussebaïle

Actinfo Chair, Institut Louis Bachelier, CREST-Polytechnique

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Motivation

- ▶ Example: a simple economy composed of $7 * 10^9$ people, each one exposed to an endowment risk distribution with 11 possible states (endowment from 0 to 10 units).

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- ▶ The main issues are that:
 - ▶ it requires to have a tremendous number of security markets.
 - ▶ it requires to make public the individual state of each person in the economy.
- ▶ What role is played by insurance companies?

Contribution

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- ▶ The presence of insurance companies allow:
 - ▶ to reduce the required number of security markets by many orders of magnitude.
 - ▶ to lower tremendously the required public information.
- ▶ I show this result in a static exchange economy with:
 - ▶ multiple commodities,
 - ▶ heterogeneous agents in terms of preferences and risk distributions,
 - ▶ no restriction on risk dependence.

Literature review

- ▶ Kihlstrom and Pauly (1971), Ellickson and Penalva-Zuasti (1997):
 - ▶ one commodity, heterogeneous agents, no restriction on risk dependence.
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- ▶ Malinvaud (1973), Cass, Chichilnisky and Wu (1996)
 - ▶ one commodity, homogeneous agents, i.i.d. risks.
 - ▶ mutual insurance companies.
- ▶ Penalva-Zuasti (2001, 2008)
 - ▶ one commodity, heterogeneous agents, i.i.d. risks.
 - ▶ stock insurance companies.

Contents

- ▶ **The model**
 - ▶ Risky economy.
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- ▶ Risky economy.
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▶ The equilibrium:

- ▶ Equilibrium.
- ▶ Pareto optimality.
- ▶ Role of insurance companies.
- ▶ Insurance premiums.

Risky economy

- ▶ Exchange economy with C commodities and N risk-averse heterogeneous agents:
 - ▶ VNM utility for agent i : $v_i(\cdot) : \mathbb{R}_+^C \rightarrow \mathbb{R}$
 - ▶ Endowment vector for agent i : $\mathbf{e}_i(s_i) = \bar{\mathbf{e}}_i - \mathbf{l}_i(s_i)$, $s_i = 1, \dots, S_i$

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An "Arrow-Debreu" state is a full specification of the individual endowments obtained by all the agents in the economy.

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Definition of an Fundamental state

A "fundamental" state is a full specification of the aggregate endowments in the economy.

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- ▶ Consumption plan of agent i in Arrow-Debreu state z denoted:
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Spot and security markets

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- ▶ T security markets:
 - ▶ security market t enables to get one ex-post money unit if t occurs, in exchange for $\sum_{z \in F_t} \pi(z)$ ex-ante money unit.
 - ▶ quantity of securities purchased/sold by agent i denoted $a_i = (a_i(1), \dots, a_i(T))$

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 - ▶ an indemnity in state z :

$$\tau_i(\mathbf{p}(z), s_i(z)) = \mathbf{p}(z) \mathbf{l}_i(s_i(z));$$

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- ▶ Share of insurer k purchased by agent i : m_{ki} .

Equilibrium

- ▶ Agent maximization problem:

$$\begin{aligned} \max_{\mathbf{x}_i, n_i, m_{ki}, a_i} \quad & \sum_z \pi(z) v_i(\mathbf{x}_i(z)) \\ \text{s.t.} \quad & \mathbf{p}(z) \mathbf{x}_i(z) = \mathbf{p}(z) \mathbf{e}_i(s_i(z)) + \tau_i(\mathbf{p}(z), s_i(z)) n_i \\ & \quad + \sum_k r_k(z) m_{ki} + a_i(t(z)), \quad \forall z \\ & \alpha_i n_i + \sum_z \pi(z) a_i(t(z)) = 0 \end{aligned}$$

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- ▶ Market clearing conditions (spot, security, stock):

$$\begin{aligned} \sum_i \mathbf{x}_i(z) &= \mathcal{E}(t(z)), \quad \forall z \in Z \\ \sum_i a_i(t) &= 0, \quad \forall t \in T \\ \sum_i m_{ki} &= 1, \quad \forall k \in M \end{aligned}$$

Pareto optimality

Result

In the decentralized economy with C spot markets, T security markets and insurance companies in competition, the equilibrium exists and the allocation is Pareto optimal.

Role of insurance companies

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- ▶ One way to reach her consumption plan for each agent i is:
 - ▶ elimination of individual endowment risk with insurance contract:
 $n_i = 1$.
 - ▶ elimination of price risk exposure and participation to aggregate risk through security and insurance stock markets:
 $a_i(t(z)) = \mathbf{p}(z)(\mathbf{x}_i(z) - \bar{\mathbf{e}}_i) + \alpha_i - \frac{1}{N} \sum_k r_k(z)$ and $m_{ki} = \frac{1}{N}$.

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$$\alpha_i = \sum_{z'} \pi(z') \mathbf{p}(z') \mathbf{l}_i(s_i(z')) = (1 + \beta_i) \bar{\mathbf{p}} \sum_{z'} \pi(z') \mathbf{l}_i(s_i(z')),$$

- ▶ in which:

$$\bar{\mathbf{p}} = \sum_{z'} \pi(z') \mathbf{p}(z')$$

$$\beta_i = \frac{1}{\bar{\mathbf{p}} \sum_{z'} \pi(z') \mathbf{l}_i(s_i(z'))} \sum_{z'} \pi(z') (\mathbf{p}(z') - \bar{\mathbf{p}}) \mathbf{l}_i(s_i(z')).$$

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- ▶ For an insurer, the higher the correlation between insured individual risks and the aggregate risk, the higher the expected profit.

Conclusion

- ▶ An exchange economy with multiple commodities, heterogeneous agents and no restriction on risk dependence.
- ▶ Only one security market per fundamental state is sufficient if there are also stock insurance companies supplying standard insurance contracts.
- ▶ Insurance and security markets respectively allow to deal with endowment risks and price risk.
- ▶ Insurance premium has an aggregate risk loading factor which is specific to each agent.